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Macroparticle model for longitudinal emittance growth caused by negative mass instability in a proton synchrotron

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Abstract

Both theoretical models and beam observations of negative mass instability (NMI) fall short of a full description of the dynamics and the dynamical effects. Clarification by numerical modeling is now practicable because of the recent proliferation of so-called computing farms. The results of modeling reported in this paper disagree with some predictions based on a long-standing linear perturbation calculation. Validity checks on the macroparticle model are described.

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I. INTRODUCTION

Most proton synchrotrons pass through an energy at which the particle circulation frequency is practically independent of momentum differences within beam bunches. At this energy, called the transition energy (E_{T}), $\frac{\partial \omega_{\mathrm{circ}}}{\partial E}$ is zero, changing from positive below E_{T} to negative above. The interparticle repulsion causes charge concentrations within the distribution to disperse below transition but to concentrate above. Just above transition energy, fluctuations in density are practically fixed in the bunch, and the particles in charge surplus regions push one another to higher and lower energy without much change in relative azimuth. If these perturbations of the smooth distribution constitute a sufficient peak current, the resulting field promotes the charge concentration so that the bunch emittance grows significantly within a few beam turns. The process has been studied with a linearized Vlasov equation for a bunch with elliptical phase space distribution plus a density modulation given by the statistical fluctuation for the number of beam particles present.[1]. Although this analytical model is simplified from the typical case for a bunched beam with its inevitable larger scale inhomogeneities, one may and has hoped that the instability threshold and the relevant Fourier components of the beam current are correctly predicted. The original Hardt paper has been studied and reviewed by Ng[2], who concludes that the analysis has assumptions in need of testing in some way. Unfortunately, detailed beam observations are difficult. There is more than one mechanism for emittance growth in crossing transition, and, although microwave components can be observed in the beam current after transition crossing[3], these observations have been made at frequencies much lower than those predicted to be dominant in NMI. There is no numerical relation to connect the observed microwave amplitudes to observed emittance growth. High frequency rf harmonics have been observed far above the Schottky background up to several GHz, implying microstructure or turbulence in the bunches.[4] Such components could be the seed for NMI observed in beam studies.

Efforts to elucidate the process by macroparticle beam models have been hampered by limits on macroparticle number, effectively limiting the bandwidth to one or two orders of magnitude below the top of the predicted range of unstable Fourier components of the beam current. The first paper using a macroparticle model to show bunch disruption at about the expected current threshold showed the disrupted distribution to be modulated by the binning frequency.[5] Later attempts with a few hundred to a few thousand times the two thousand macroparticles used by Lee and Teng in the original effort also evidenced this dubious property. In a recent paper by this author[6], results which did show clustering at less than the binning frequency evidenced some disagreement with the analytic model. However, despite a macroparticle count of $1.2 \cdot 10^7$, the calculation was manifestly statistics limited, and only qualitative conclusions were justified. The ultimate macroparticle model would employ a test particle for each particle in the real bunch. Despite the rapid increase in computing resources available for routine research, this ideal is unlikely to be attained for some years. This paper reports results using at most $6.4 \cdot 10^8$ macroparticles to simulate a bunch in the Fermilab Main Injector with intensity ranging from $6 \cdot 10^{10}$ to $2 \cdot 10^{11}$ protons in 0.2 eVs. Because these calculations are also statistically marginal, tests have been made to establish the validity of quantitative results.

II. BEAM PROPERTIES AND ACCELERATOR PARAMETERS

Most of the examples reported relate to properties of the Fermilab Main Injector (FMI) which is characterized for present purposes by the parameters in Table I. However, for a comparison to the Hardt model I have used parameters and results for the old Fermilab Main Ring (MR) from ref. 3, also included in Table I. As may be seen, the two rings are not so very different despite the difference in circumference.

III. VALIDATION OF THE MACROPARTICLE MODEL

Taking the specific instance of a $4 \cdot 10^{10}$ proton bunch in the MR, the macroparticle model shows a factor of 2.0 emittance growth at 0.1 eVs but just 4.6% growth at 0.12 eVs. This outcome is consistent with Ng's evaluation of Hardt's c parameter as 1.31 in the first case and 0.84 in the second because the value c=1 is the threshold for NMI according to Hardt's analysis.

Hardt's paper is very terse and perhaps difficult to understand for other reasons also. One of the apparent mysteries is why the threshold seems so sensitive with respect to bunch emittance. Fortunately there is an alternative, semi-phenomenological, means to establish the threshold and understand the sensitivity intuitively. Assume that any perturbation of the particle flow caused by a full micro bucket entirely contained within the bunch will be negligible; for a uniform distribution this is certainly true. Then a plausible threshold criterion is

$$H_{\mu\mathrm{bucket}} \geq H_b \quad , \tag{1}$$

TABLE I: Ring parameters used in the models

Parameter	Symbol	FMI	MR	Units
Circumference	C	3319.42	6283.19	m
transition energy/$m_\circ c^2$	γ_T	21.84	18.85	
rf peak voltage	$V_{\rm rf}$	3.7	2.5	MV
rf harmonic	h	588	1113	
beam circulation frequency	$f_\circ$	90.2195	47.6478	kHz
ramp rate	$\dot{\gamma}$	240.0	109.94	s^{-1}
synchronous phase	$\phi_{\rm s}$	42.38	60.00	deg
nonadiabatic time	$T_{\rm na}$	1.76	3.00	ms
beampipe radius	b	2.5	3.5	cm
beam radius	a	0.4	0.4	cm
geometric factor	$g_\circ$	4.66	4.89	
harmonic of $f_\circ$ for $g = \frac{1}{2}g_\circ$	$n_{1/2}$	2238395	3312217	
number of protons per bunch	N	0.6 – $2.0 \cdot 10^{11}$	$0.4 \cdot 10^{11}$	
average bunch current	$\bar{I}_{\rm bunch}$	0.51 – 1.7	0.34	A

where $H_{\mu\rm bucket}$ is the height of the micro bucket generated by the current fluctuations and H_b is the bunch height. Because the micro bucket height depends on V/h and the space charge impedance is proportional to h, no harmonic need be specified. The least obvious parameter is the relevant current, which is basically the expected amplitude of the Schottky current for the average beam current divided by the bunching factor to account for the circumstance that the instability occurs at the highest current region in the bunch.:

$$\hat{I}_{\rm bunch} = \frac{2\bar{I}_{\rm bunch}}{B\sqrt{N}} \quad . \tag{2}$$

Everything needed to evaluate this criterion is available in texts and handbooks of accelerator or beam physics. If one could also infer the dominant harmonics for the instability, this approach could qualify as an explanation of NMI. The growth rate of the amplitudes (the synchrotron frequency in the micro buckets) increases linearly with frequency, so very high frequency modes should dominate. The so called geometric factor g is reduced with frequency to account approximately for the frequency dependence of the boundary conditions in the beam chamber according to Hardt's formula:[1]

$$g(n) = \frac{g_\circ}{1 + (n/n_{1/2})^2} \ \ , \tag{3}$$

where

$$g_\circ = 1 + 2\ln(b/a) \ \ , \tag{4}$$

$$n_{1/2} = \gamma R_{\rm eq} \left(\frac{1.6}{b} + \frac{0.52}{a} \right) \ \ , \tag{5}$$

and n is the harmonic number with respect to the beam circulation frequency, $R_{\rm eq}$ is the mean orbit radius, and a and b are beampipe and mean beam radii respectively. Thus, the upper end of the active band of harmonics is limited at some point by the decrease in g.

Notice that for g constant, all harmonics are unstable and develop the same micro bucket height. The higher harmonics dominate because they grow faster. A practical benefit of this fact is that tracking with too few particles and too few bins gives approximately the correct emittance growth over somewhat longer time — still only a few beam turns. Therefore, if one wants to evaluate the emittance growth to be expected in some particular case, it is not necessary to employ hundreds of millions of macroparticles. A single processor can typically handle 10^7 macroparticles. The number of bins should be fixed so that a reduction in the number of macroparticles by, say, a factor of two does not change the result within the desired precision. Something like 1024 bins should be possible. Another test, which applies at the maximum macroparticle number, is to vary the seed for a random distribution.

The self-bunching model stability criterion is consistent with Ng's evaluation of NMI in the MR and also with the FMI examples reported below.

Certainly the consistency of the macroparticle model with the analysis is reassuring, but it is almost surprising in view of the analytical assumption of pure Schottky noise whereas just $6.4 \cdot 10^8$ macroparticles represent a bunch of $4 \cdot 10^{10}$ protons in the numerical model. Given that both models have approximations, some further tests are desirable. It is also desirable that the macroparticle model should be validated as much as possible independent of comparisons with the analytical Schottky noise model because one wants to argue that differing results are evidence of the approximations in the analytical treatment.

The macroparticle model assumes an elliptical phase space distribution, which has a parabolic projection along the beam direction. A parabolic number distribution of N particles with width $2w$ can be expressed as

$$n(z) = \frac{3N}{4w}[1 - (z/w)^2] \ \ . \tag{6}$$

The charge distribution of protons is just $\lambda(z) = en(z)$ and the self field of the bunch with perfectly conducting wall boundary conditions is

$$E_{\mathrm{sc}} = -\frac{g}{4\pi\varepsilon_\circ\gamma^2}\frac{\partial\lambda}{\partial z} \quad . \tag{7}$$

By restricting the bandwidth of the numerical calculation to $\mathcal{O}(100)$ harmonics, only the field from the bunch envelope is calculated; this can be compared to the result of eq. 7. The restricted bandwidth calculation of the field of a 0.2 eVs bunch of $2 \cdot 10^{11}$ protons was repeated with $6.4 \cdot 10^8$, $8 \cdot 10^7$, 10^7, and $1.25 \cdot 10^6$ macroparticles to assure that the slope of the bunch envelope was precisely determined. The bunch width was taken from the computed charge histogram. The voltage of ± 133 kV given by the model agrees to the given number of figures with the voltage calculated by multiplying the field from eq. 7 by the ring circumference. The code has the capability of calculating the space charge voltage in either frequency or time domain; these largely independent calculations also agree well. Furthermore, the parallelized code used for this modeling agrees with the ESME code[7], which has had several years of shake down in a wide range of applications.

The results of the restricted bandwidth calculation using 256 bins was the same for macroparticle numbers from $6.4 \cdot 10^8$ to 10^7 but differed somewhat for $1.25 \cdot 10^6$. Jie Wei has shown for the space charge potential that the number of macroparticles should be cubed if the number of bins is doubled in order that the statistical noise per bin remain the same.[8] From this n-cubed binning rule one can infer that $6.4 \cdot 10^8$ macroparticles is also adequate for 2048 bins. The FMI NMI model, however, should have 4096 or more bins to cover the range of Fourier amplitudes predicted to be important by the Schottky noise model. Given the choice of limiting the bandwidth and repeating the history of producing instability at the binning frequency or accepting numerical noise that is possibly too high by $\sqrt{8}$, there is no question that it is more interesting to consider the wider bandwidth. The observation that the $1.25 \cdot 10^6$ macroparticle restricted bandwidth calculation differed from those with higher statistics by about one percent on the calculated voltage, suggests that the final factor of two in the binning should not entirely vitiate the results. Another reassuring observation is the agreement of frequency domain and time domain results with 4096 bins per rf period. In addition, results produced at 4096 bins by varying the seed for the random number generator agreed within a few percent on emittance growth and strength of the larger Fourier amplitudes.

The checks described are offered as evidence that not only can the macroparticle model be prudently applied for guidance in practical considerations but also it is suitable for examining predictions derived from the analytical Schottky noise model. Both of these points of view will be

TABLE II: Emittance growth for several FMI cases

Initial emittance $[\mathrm{eVs}]$	Bunch population $[\times 10^{10}]$	$\dot{\gamma}$ $[\mathrm{s}^{-1}]$	Final emittance $[\mathrm{eVs}]$	$f_{\max}$ [GHz]	f_{worst} [GHz]
0.2	6.0	240	0.205	87	56
0.2	10.0	240	0.209	97	46
0.2	20.0	240	0.411	97	39
0.2	10.0	120	0.221	93	56
0.2	20.0	120	0.851	97	46

adopted in specific cases applying to the FMI.

IV. APPLICATION TO THE FERMILAB MAIN INJECTOR

Generally it is easier to negotiate transition crossing if the longitudinal emittance is small so that single particle nonlinear effects are minimized. A major caveat is that NMI can disrupt the bunch far more than the nonlinearities generally do. Thus, in working out longitudinal impedance budgets one wants to minimize the bunch area approaching transition with the constraint of remaining above the threshold for NMI. Coupled bunch instability stands out as a collective effect that could be exacerbated by small, bright bunches; therefore the capabilities of damper systems are an additional consideration. The NMI threshold, however, is a brick wall lower limit on emittance.

The principal practical outcome of the modeling is summarized in the growth factor *vs.* initial bunch emittance plotted in Fig. 1. The initial parameters are those of a $2 \cdot 10^{11}$ proton bunch in the FMI. The final emittance is the value just after the nonadiabatic time following transition. The predicted threshold emittance (at fixed intensity) is 0.25 eVs. From this plot one can see that NMI is not threatening the present operation at about $6 \cdot 10^{10}$ protons per bunch at 0.2 eVs pre-transition bunch emittance. Better performance in routine operation might be obtained by measures to reduce the bunch area approaching transition to reduce further emittance dilution during transition crossing.

There are two predictions of the analytical Schottky noise model besides the stability threshold which can be rather readily compared to macroparticle model results, *viz.*, the frequency of fastest amplitude growth and the frequency of maximum integrated harmonic growth. In the tabulated

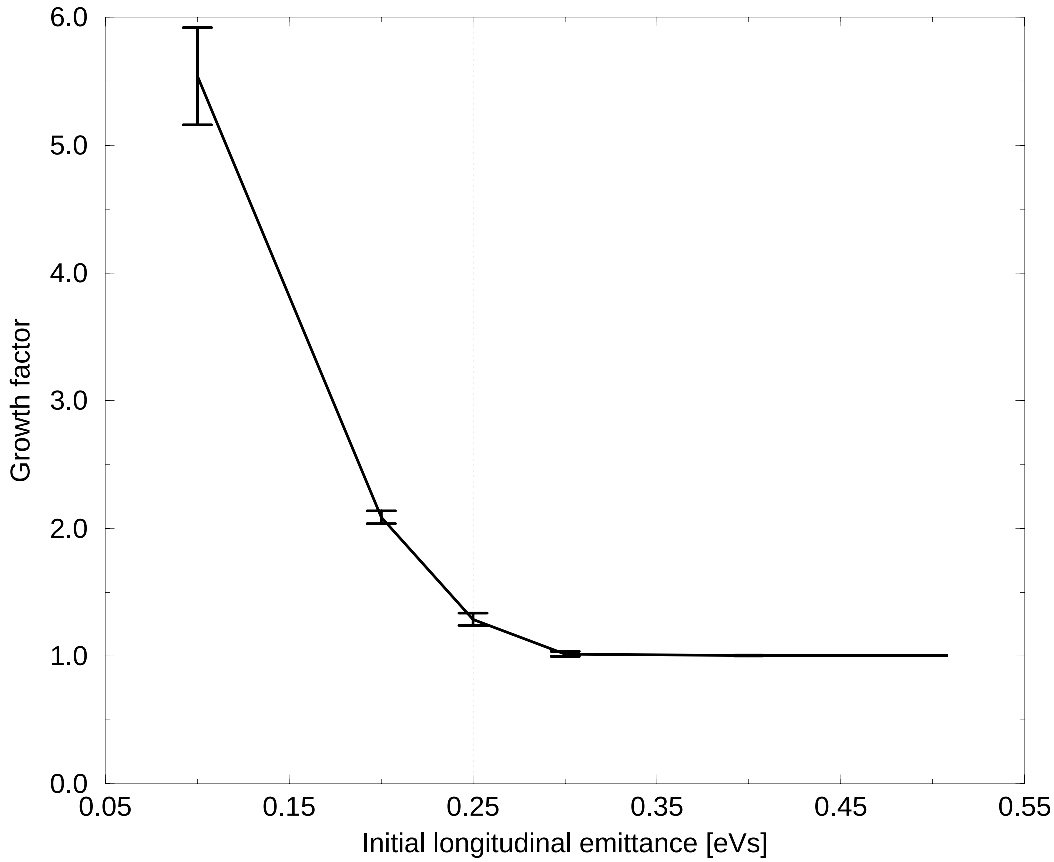

FIG. 1: Emittance growth factor *vs*. initial emittance [eVs] for bunches of $2 \cdot 10^{11}$ protons crossing transition in the FMI. Each point is determined by a tracking with $6.4 \cdot 10^8$ macroparticles and a run with 10^8. The statistically weighted mean is the central value with error bars given by the difference between the two runs. The vertical dashed line is the NMI threshold evaluated either from Hardt's formulas or the self-bunching criterion.

results, Table II, the last two columns contain approximations $f_{\rm max}$ and $f_{\rm worst}$ of these frequencies estimated visually from plots of the rms amplitudes *vs*. time. These estimates are crude, but they consistently agree that the most active frequencies are somewhat lower than predicted by the analytical model which gives $f_{\rm max} = 117$ GHz and $f_{\rm worst} = 67$ GHz in all of these cases. The same kind of disagreement was noted also in ref. [6]. In that paper it was further noted that the important frequencies become lower as the instability develops. Therefore, the lower values probably are the result of the nonlinear equations of motion in the macroparticle model.

V. POTENTIAL REFINEMENTS

There are some tactics for putting the comparison between analysis and macroparticle models on a firmer basis. The most obvious is simply to increase the number of macroparticles. Thirty-two fast PC nodes were used for most of the reported results. A significant improvement in statistics would require something like ten times this number. A more modest approach is to use a distribution based on a pseudo-random sequence, such as the Sobel sequence. It would become noisier because of the effects of synchrotron oscillation, but by starting the tracking just a few oscillation periods before transition the distribution would still be quieter than a quasi-random distribution of the same number of macroparticles. The strength of the amplitudes can be measured so that the distribution can be fine-tuned for the correct Schottky noise amplitudes at transition. If needed, some pre-transition perturbation of the bunch can be avoided by introducing the space charge force gradually. Some such approach or combination of approaches could reduce the numerical noise sufficiently to offer a strong check on the 4096 bin results. It is by no means apparent, however, that such a test is required to support the reported results, nor is it apparent that fine tuning the distribution is entirely free of bias. Another possibility would be to improve the determination of f_{max} and f_{worst} by numerical analysis to permit a more rigorous comparison of the macroparticle results to the analytical values. This latter improvement would have no utility in practical applications of the model.

[1] W. Hardt, in $9^{\underline{\mathrm{th}}}$ *Int. Conf. High Energy Accel.* (SLAC, Stanford, 1974), pp. 434–438.

[2] K.-Y. Ng, Physics Note Fermilab-FN-0713, Fermi National Accelerator Laboatory, Batavia IL (2001).

[3] I. Kourbanis and K.-Y. Ng, in *1993 IEEE Part. Acc. Conf.* (IEEE, Washington, DC, 1993), pp. 3630–3632.

[4] G. Jackson et al., in *1991 IEEE Part. Acc. Conf.* (IEEE, San Francisco CA, 1991), pp. 1758–1760.

[5] W. W. Lee and L. C. Teng, in $8^{\underline{\mathrm{th}}}$ *Int. Conf. High Energy Acc.* (CERN, Geneva, 1971), pp. 327–330.

[6] J. A. MacLachlan, in $20^{\underline{\mathrm{th}}}$ *ICFA Adv. Beam Dyn. Workshop*, Fermi National Accelerator Laboratory (American Inst. of Phys., 2002), vol. 642 of *AIP Conf. Proc.*, pp. 68–70.

[7] J. A. MacLachlan, in $17^{\underline{\mathrm{th}}}$ *Int. Conf. High Energy Acc.* (Joint Institute for Nuclear Reseaarch, Dubna, 1998).

[8] J. Wei, Ph.D. thesis, State University of New York (Stony Brook) (1990).